

Analysis of the Lee-Yang zeros in a dynamical mass generation model in three dimensions

I.M. Barbour^a, W. Franzki^b and N.Psycharis^a *

^aDept. of Physics and Astronomy, University of Glasgow G12 8QQ, U.K., (UKQCD Collaboration)

^bInstitute of Theoretical Physics E, RWTH Aachen, D-52056 Aachen, GERMANY

We investigate a strongly U(1) gauge theory with fermions and scalars on a three dimensional lattice and analyze whether the continuum limit might be a renormalizable theory with dynamical mass generation. Most attention is paid to the weak coupling region where a possible new dynamical mass generation mechanism might exist. There we investigate the mass of the composite fermion, the chiral condensate and the scaling of the Lee-Yang zeros.

1. Introduction

The fermion-gauge-scalar model ($\chi U\phi$ model), has been suggested as a model for dynamical mass generation in four dimensions in [2,4]. Investigating such a lattice model in three dimensions [1] we find three regions in the $\beta-\kappa$ plane with possibly different critical behaviour in the chiral limit of the model. These three regions are presented in Fig.(1). The region X of the phase diagram is conceivably analytically connected with either the Nambu or Higgs phase but it may well be a separate phase. In this region the measured fermion mass is large but the chiral condensate is very small (zero within our numerical accuracy). We also investigated the scaling of the Lee-Yang zeros in this interesting region of the phase diagram in order to clarify the situation.

2. The model

The $\chi U\phi_3$ model is defined by the lattice action:

$$S_{\chi U\phi} = S_\chi + S_U + S_\phi, \quad (1)$$

with

$$S_\chi = \frac{1}{2} \sum_x \bar{\chi}_x \sum_{\mu=1}^3 \eta_{\mu x} (U_{x,\mu} \chi_{x+\mu} - U_{x-\mu,\mu}^\dagger \chi_{x-\mu})$$

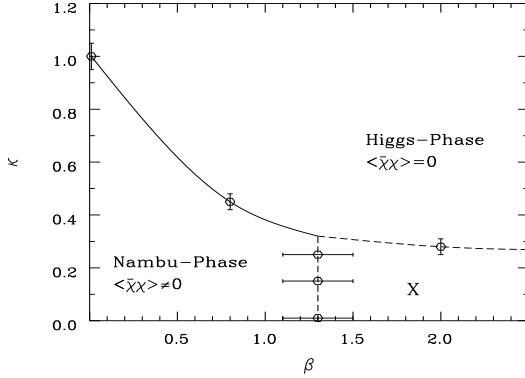


Figure 1. Phase diagram of the $\chi U\phi_3$ model for $m_0 = 0$. All phase transitions seem to be 2nd order.

$$\begin{aligned} &+ am_0 \sum_x \bar{\chi}_x \chi_x, \\ S_U &= \beta \sum_{x,\mu<\nu} (1 - \text{Re } U_{x,\mu\nu}), \\ S_\phi &= -\kappa \sum_x \sum_{\mu=1}^3 (\phi_x^\dagger U_{x,\mu} \phi_{x+\mu} + \text{h. c.}) . \end{aligned}$$

χ_x are the Kogut-Susskind fermion fields. We are interested in the chiral limit $m_0 = 0$.

2.1. Observables

In order to investigate the chiral properties of the model we concentrate first on two observables:

*Talk presented by N.Psycharis

the chiral condensate and the fermion mass[1].

The third observable we concentrate on are the Lee-Yang zeros. The canonical partition function of the system can be written as[3,1]:

$$Z(\beta, \kappa, am_0) = \sum_{n=0}^{\frac{V}{2}} A_n[\beta, \kappa](am_0)^{2n}. \quad (2)$$

The Lee-Yang zeros are the zeros of this polynomial representation of the partition function. The errors in the Lee-Yang zeros are estimated by a Jackknife method.

The critical properties of the system are determined by the zeros lying closest to the real axis. y_1 is the zero with the smallest imaginary part and is called edge singularity. With increasing finite volume it converges to the critical point. For a continuous phase transition the position of the zeros closest to the real axis in the complex plane is ruled by the scaling law

$$y_i(\beta, \kappa, L) - y_R(\beta, \kappa, \infty) = A_i L^{-1/s} \quad (3)$$

where A_i 's are complex numbers and the exponent $s = s(\beta, \kappa)$ [1] describes the finite size scaling of the correlation length. It follows that the real and the imaginary part of the zeros should scale independently with the same exponent.

This scaling law can also be extended to the case of a first order phase transition. In this case, for a three-dimensional model we expect $s = \frac{1}{3}$.

Far away from the critical point we expect linear scaling[1] in the log-log plot with $s = 1/3$ in the broken phase and $s = 1$ in the symmetric phase. At the critical point itself we expect linear scaling.

3. Universality at Strong Coupling

An investigation of the scaling of the Lee-Yang zeros, the chiral condensate and the fermion mass at $\beta = 0$ and $\beta = 0.80$, presented in[1], shows that the chiral phase transition of the $\chi U\phi_3$ model is in the same universality class at both these β values. At $\beta = 0$ the scalar and gauge fields can be integrated out exactly and we end up with a lattice version of the GN_3 model. This model is known to have a chiral phase transition which is nonperturbatively renormalizable.

It is also very likely that the same universality class holds also for $0 \leq \beta \leq 0.80$.

4. Weak coupling region

We first look at the neutral fermion mass and the chiral condensate in the weak coupling region.

The mass of the neutral fermion is large for $\kappa < 0.27$ and it becomes small for larger κ at $\beta = 2.0$ [1]. For $\kappa > 0.27$ its value probably vanishes in the infinite lattice size limit.

The condensate show a crossover behaviour at $\kappa \approx 0.27$ with

$$\langle \bar{\chi}\chi \rangle(am_0)_{\kappa < 0.27} < \langle \bar{\chi}\chi \rangle(am_0)_{\kappa > 0.27}$$

also at $\beta = 2.0$ [1]. However, at large κ we believe that we are in the Higgs phase where the condensate is zero in the chiral limit. Therefore, in this limit, the condensate seems to be zero for all κ at $\beta = 2.0$.

At $\kappa = 0.25$ and at various β , the condensate is large at small β but becomes very small(zero) in the chiral limit for $\beta > 1.5$. The fermion mass decreases for increasing β but then stabilizes with $am_F > 1$. So it is clearly non zero at all β .

It is surprising but also interesting that the fermion mass is different from zero with unbroken chiral symmetry in the region X of the phase diagram.

4.1. The Lee-Yang zeros at Weak Coupling

In order to clarify the situation in the X region of the phase diagram we investigated the Lee-Yang zeros.

The edge singularity y_1 has a nonzero real part in this region compare to the other regions of the phase diagram where the zeros have zero real part. The zeros appear in conjugate pairs. We define the edge singularity in this region to be the zero with smallest positive imaginary part and count each pair only once.

Fig. 2 shows the behaviour of the imaginary part of the edge singularity as a function of lattice size at $\beta = 2.00$ and various κ . We see that the exponent s seems to be universal for $\beta = 2.00$ and small κ values, with a value of $s \approx 0.7$.

Figs 3 and 4 show the finite size scaling of the imaginary and the real part of the lowest zeros

for $\beta = 2.00$ and $\kappa = 0.00$, respectively. This is a point in the the region X. The first two zeros have, within the numerical precision, identical imaginary part but their real parts differ by a factor of about 3.5. Their imaginary parts scale linearly in the log-log plot with the same exponent s within the numerical precision. Although with somewhat larger errors, the same seems to happen also for their real parts. This pattern of zeros can be found everywhere in the region X.

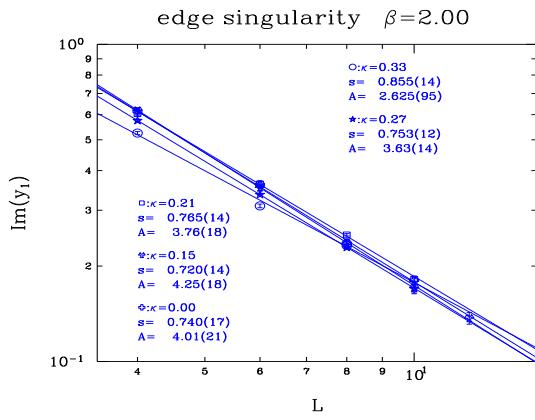


Figure 2. Imaginary part of y_1 as function of the lattice size for different κ at $\beta = 2.00$.

5. Conclusions

The results in the weak coupling region, and specially in the interesting X region, show that the X region could belong to the Nambu phase if the condensate is really nonzero but unmeasurably small. It could also belong to the Higgs phase if the charged fermion becomes massless in the chiral limit and is no longer a bound state. Finally there is also the explanation that it could be a separate phase with a new dynamical mass generation mechanism. The Lee-Yang zeros now distinguish the region X from the Nambu and Higgs phase by a nonvanishing real part of the lowest zeros and a scaling, which can neither be described by an exponent $s = 1/3$ nor $s = 1$.

6. Acknowledgements

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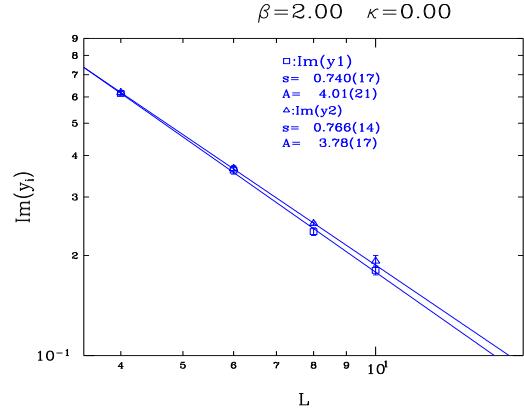


Figure 3. Imaginary part of the first two Lee Yang zeros for $\beta = 2.00$, $\kappa = 0.00$ as function of lattice size.

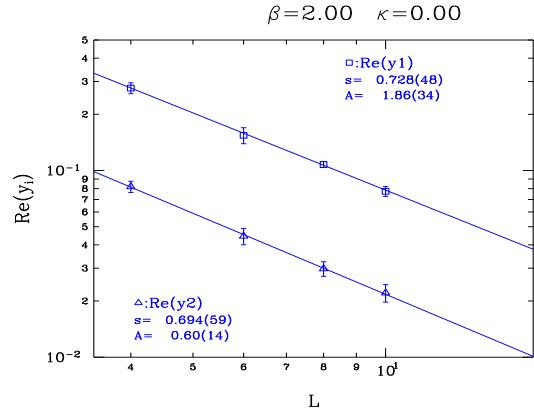


Figure 4. Real part of the first two Lee Yang zeros for $\beta = 2.00$, $\kappa = 0.00$ as function of lattice size.

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